Practice §16.2-3: Fundamentals of Line Integrals

Instructions: Complete each of the following exercises for practice.

1. Compute the line integral $\int_C f\ ds$.

   (a) $f(x, y) = y$;
   C: $\langle t^2, 2t\rangle$ for $0 \leq t \leq 3$

   (b) $f(x, y) = \dfrac{x}{y}$;
   C: $\langle t^3, t^4\rangle$ for $1 \leq t \leq 2$

   (c) $f(x, y) = xy^4$;
   C: the right half of the circle $x^2 + y^2 = 16$

   (d) $f(x, y) = xe^y$;
   C: the line segment from $(2, 0)$ to $(5, 4)$

   (e) $f(x, y, z) = x^2y$;
   C: $\langle\cos(t), \sin(t), t\rangle$ for $0 \leq t \leq \frac{\pi}{2}$

   (f) $f(x, y, z) = x^2 + y^2 + z^2$;
   C: $\langle t, \cos(2t), \sin(2t)\rangle$ for $0 \leq t \leq 2\pi$

   (g) $f(x, y, z) = y^2z$;
   C: the line segment from $(3, 1, 2)$ to $(1, 2, 5)$

   (h) $f(x, y, z) = x\exp(yz)$;
   C: the line segment from $(0, 0, 0)$ to $(1, 2, 3)$

2. Compute the indicated line integral directly.

   (a) $\int_C (x^2y + \sin(x))\ dy$;
   C: the arc of $y = x^2$ from $(0, 0)$ to (π, π^2)

   (b) $\int_C e^x\ dx$;
   C: the arc of $y = x^3$ from $(-1, -1)$ to $(1, 1)$

   (c) $\int_C xye^{yz}\ dy$;
   C: $\mathbf{r}(t) = \langle t, t^2, t^3\rangle$ for $0 \leq t \leq 1$

   (d) $\int_C y\ dx + z\ dy + x\ dz$;
   C: $\mathbf{r}(t) = \langle\sqrt{t}, t, t^2\rangle$ for $1 \leq t \leq 4$

   (e) $\int_C z^2\ dx + x^2\ dy + y^2\ dz$;
   C: the segment from $(1, 0, 0)$ to $(4, 1, 2)$

3. Determine whether or not $\mathbf{F}$ is a conservative vector field. If it is, compute a potential function for $\mathbf{F}$.

   (a) $\mathbf{F} = \langle xy + y^2, x^2 + 2xy\rangle$

   (b) $\mathbf{F} = \langle y^2 - 2x, 2xy\rangle$

   (c) $\mathbf{F} = \langle y^2e^{xy}, (1 + xy)e^{xy}\rangle$

   (d) $\mathbf{F} = \langle ye^x, e^x + e^y\rangle$

   (e) $\mathbf{F} = \langle ye^x + \sin(y), e^x + x\cos(y)\rangle$

   (f) $\mathbf{F} = \langle y^2\cos(x) + \cos(y), 2y\sin(x) - x\sin(y)\rangle$

   (g) $\mathbf{F} = \left\langle \ln(y) + \dfrac{y}{x}, \ln(x) + \dfrac{x}{y}\right\rangle$

4. Use the Fundamental Theorem of Line Integrals to evaluate $\int_C \mathbf{F}\cdot\ d\mathbf{r}$.

   (a) $\mathbf{F} = \langle 3 + 2xy^2, 2x^2y\rangle$
   C: the arc of $y = \dfrac{1}{x}$ from $(1, 1)$ to $(4, \frac{1}{4})$

   (b) $\mathbf{F} = \langle x^2y^3, x^3y^2\rangle$
   C: $\mathbf{r}(t) = \langle t^3 - 2t, t^3 + 2t\rangle$ for $0 \leq t \leq 1$

   (c) $\mathbf{F} = \langle(1 + xy)e^{xy}, x^2e^{xy}\rangle$
   C: $\mathbf{r}(t) = \langle\cos(t), 2\sin(t)\rangle$ for $0 \leq t \leq \frac{\pi}{2}$

   (d) $\mathbf{F} = \langle xy, xz, xy + 2z\rangle$
   C: the segment from $(1, 0, -2)$ to $(4, 6, 3)$

   (e) $\mathbf{F} = \langle y^2z + 2xz^2, 2xyz, xy^2 + 2x^2z\rangle$
   C: $\mathbf{r}(t) = \left\langle\sqrt{t}, t + 1, t^2\right\rangle$ for $0 \leq t \leq 1$

   (f) $\mathbf{F} = \langle yze^{xz}, e^{xz}, xye^{xz}\rangle$
   C: $\mathbf{r}(t) = \langle t^2 + 1, t^2 - 1, t^2 - 2t\rangle$ for $0 \leq t \leq 2$

   (g) $\mathbf{F} = \langle\sin(y), x\cos(y) + \cos(z), -y\sin(z)\rangle$
   C: $\mathbf{r}(t) = \langle\sin(t), t, 2t\rangle$ for $0 \leq t \leq \frac{\pi}{2}$